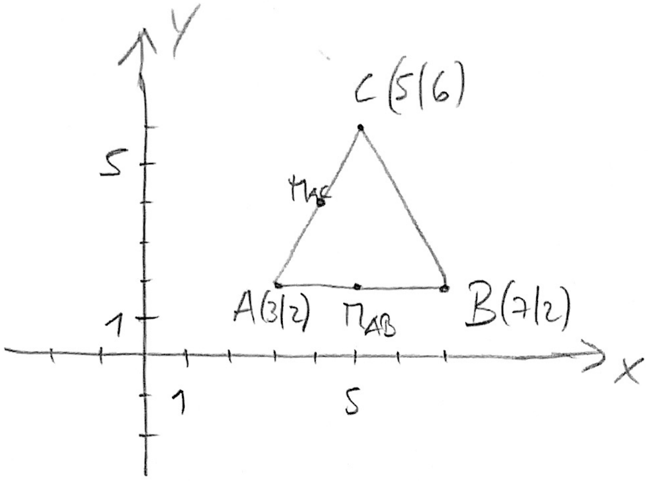
Lösungen zu AnGeo 10, II



1) $G\left(\frac{3+7+5}{3} \middle| \frac{2+2+6}{3}\right)$

$= \underline{\left(5 \middle| \frac{10}{3}\right)}$

2) Umkreismittelpunkt Ω ist der Schnittpunkt der Mittelsenkrechten m des ΔABC:

$\Rightarrow m_{AB}$: $\vec{AB} = \vec{n}_{m_{AB}} = \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$ m_{AB}: $4x + c = 0$, $M_{AB}(5|2)$ einsetzen:

$4 \cdot 5 + c = 0 \Leftrightarrow c = -20 \Rightarrow \underline{x - 5 = 0}$

$\Leftrightarrow \underline{x = 5}$

$\Rightarrow m_{AC}$: $\vec{AC} = \vec{n}_{m_{AC}} = \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$ m_{AC}: $x + 2y + c = 0$

$M_{AC}(4|4)$ einsetzen: $4 + 2 \cdot 4 + c = 0 \Leftrightarrow c = -12$

$\Rightarrow m_{AC}$: $x + 2y - 12 = 0$

$\Rightarrow \Omega = m_{AC} \cap m_{AB}$: $5 + 2y - 12 = 0 \Leftrightarrow \underline{y = \frac{7}{2}}$ $\Rightarrow \underline{\underline{\Omega\left(5 \middle| \frac{7}{2}\right)}}$

$R = \|\vec{A\Omega}\| \; (= \|\vec{B\Omega}\| = \|\vec{C\Omega}\|)$,

$= \left\|\begin{pmatrix}2\\\frac{3}{2}\end{pmatrix}\right\| = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$ $\Rightarrow \boxed{R = \frac{5}{2}}$

3) Der Höhenschnittpunkt ist der Schnittpunkt der Höhenlinien des ΔABC:

$\Rightarrow h_B$: $x + 2y + c = 0$, $B \in h_B \Rightarrow 7 + 2 \cdot 2 + c = 0 \Leftrightarrow c = -11$

$\hookrightarrow h_B$: $\underline{x + 2y - 11 = 0}$

$\Rightarrow h_C$: $\underline{x = 5}$

$\Rightarrow H = h_B \cap h_C$: $5 + 2y - 11 = 0 \Leftrightarrow \underline{y = 3}$ $\rightsquigarrow \underline{H(5|3)}$

a) $\rightarrow (AB)$: $\vec{AB} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} = \begin{pmatrix} b \\ -a \end{pmatrix} \Rightarrow (AB): \; 6x + c = 0 \Rightarrow c = -42$

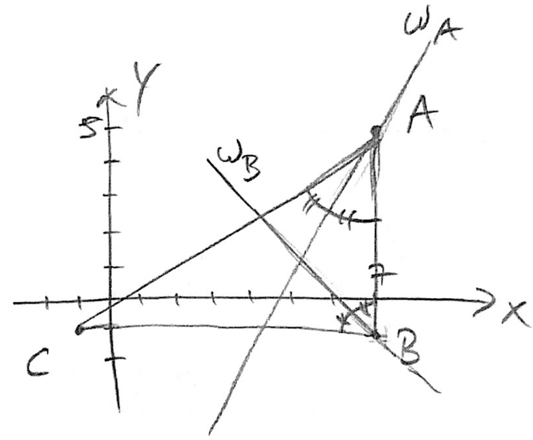
$\hookrightarrow (AB): \; 6x - 42 = 0 \Leftrightarrow x - 7 = 0$

$\rightarrow (AC)$: $\vec{AC} = \begin{pmatrix} -8 \\ -6 \end{pmatrix} = \begin{pmatrix} b \\ -a \end{pmatrix}$

$\hookrightarrow (AC): \; 6x - 8y + c = 0 \Rightarrow c = -2$

$\hookrightarrow (AC): \; 6x - 8y - 2 = 0$

$\Leftrightarrow 3x - 4y - 1 = 0$



$$\delta(P;(AB)) \stackrel{!}{=} \delta(P;(AC))$$

$$\frac{x-7}{1} = \pm \frac{3x-4y-1}{\sqrt{25}}$$

$$\frac{x-7}{1} = \pm \frac{3x-4y-1}{5}$$

$\oplus$:

$5(x-7) = 3x - 4y - 1$

$5x - 35 = 3x - 4y - 1$

$-4y = -2x + 34$

$y = -\frac{1}{2}x + \frac{2}{17}$

$\ominus$:

$5(x-7) = -(3x - 4y - 1)$

$5x - 35 = -3x + 4y + 1$

$4y = 8x - 36$

$w_A: y = 2x - 9$ vgl. Skizze!

$\rightarrow (BC)$: $\vec{BC} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} = \begin{pmatrix} b \\ -a \end{pmatrix} \Rightarrow (BC): \; -8y + c = 0 \Rightarrow c = -8$

$\hookrightarrow (BC): \; -8y - 8 = 0 \Leftrightarrow y + 1 = 0$

$$\delta(P;(AB)) \stackrel{!}{=} \delta(P;(BC))$$

$$\frac{x-7}{1} = \pm \frac{y+1}{1}$$

$$x - 7 = \pm(y+1)$$

$\oplus$:

$x - 7 = y + 1$

$y = x - 8$

$\ominus$:

$x - 7 = -(y+1) = -y - 1$

$w_B: y = -x + 6$ vgl. Skizze!

b) $\omega_A \cap \omega_B = I \Rightarrow 2x - 9 = -x + 6$

$3x = 15 \Leftrightarrow x = 5 \Rightarrow y = 1$

Einsetzen in ω_A oder ω_B.

$\Rightarrow \underline{\underline{I(5|1)}}$

c) $R = \delta(I;(AC)) \left[= \delta(I;(AB)) = \delta(I;(BC)) \right]$

$\Downarrow$

$\delta(I;(AC)) = \frac{|3 \cdot 5 - 4 \cdot 1 - 1|}{\sqrt{25}} = \frac{10}{5} = \boxed{2 = R}$