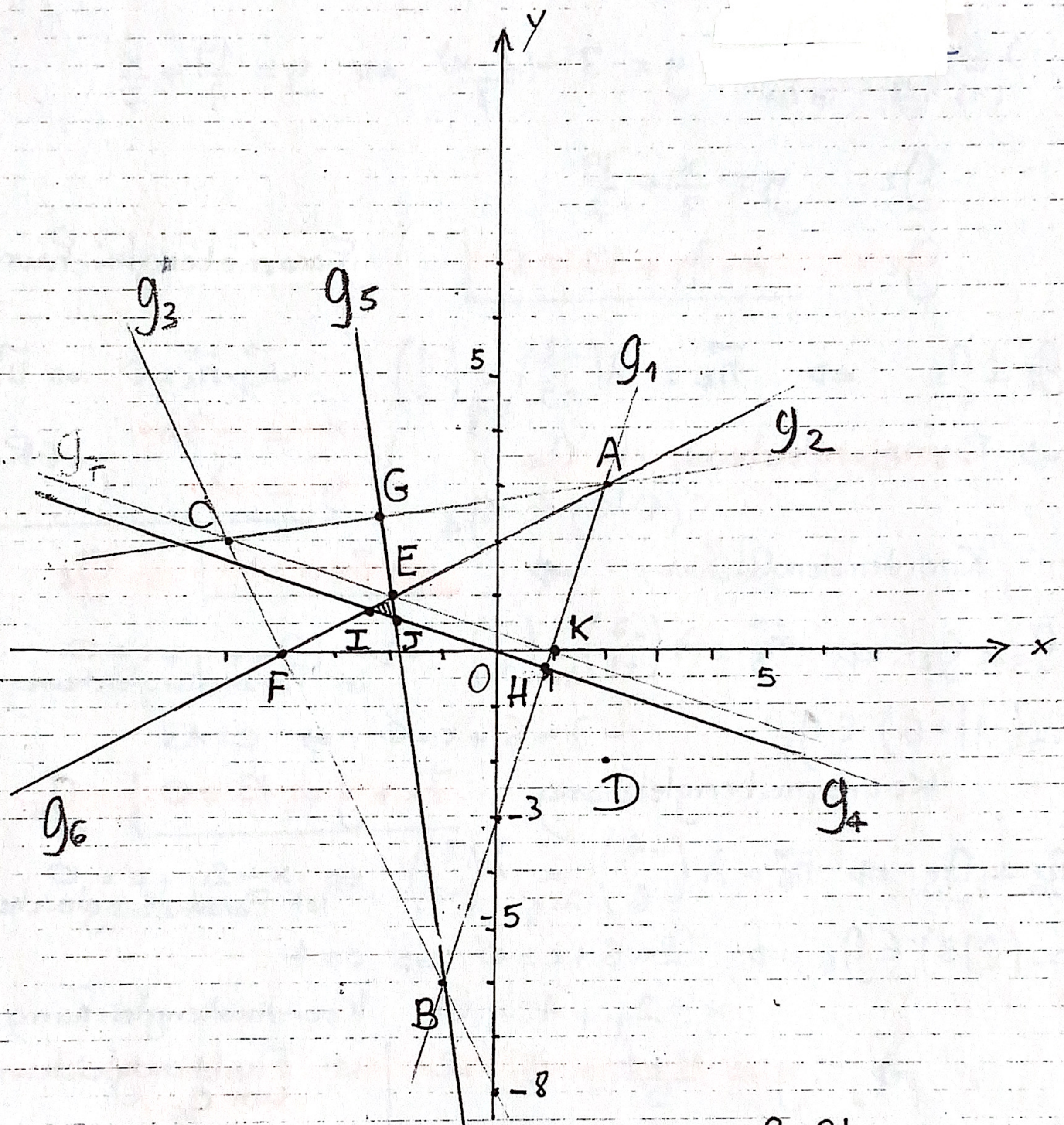
Musterlösung zu Analytische Geometrie 8, 2MSB




**1.1** $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$ ist Richtungsvektor

$g_1 = (AB): \begin{cases} x = 2 - 3t & (1) \\ y = 3 - 9t & (2) \end{cases} \quad t \in \mathbb{R}$

Lösung des Systems durch Eliminieren des Parameters $t$ (aus (1)): $t = \frac{2-x}{3}$

$$y = 3 - 9 \cdot \left(\frac{2-x}{3}\right) = 3 - 3(2-x) = -3 + 3x$$

$g_1$: $y = 3x - 3$ Funktionsgleichung

$g_1$: $3x - y - 3 = 0$ Koordinatengleichung

$\overrightarrow{BC} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$ ist Richtungsvektor

$g_3 = (BC): \begin{cases} x = -1 - 4t & (1) \\ y = -6 + 8t & (2) \end{cases} \quad t \in \mathbb{R}$

$t \underset{(1)}{=} \frac{-1-x}{4} \qquad y = -6 + 8\left(\frac{-1-x}{4}\right) = -6 + 2(-1-x) = -8 - 2x$

$g_3$: $y = -8 - 2x = -2x - 8$

$g_3$: $2x + y + 8 = 0$. Koordinatengleichung

$g_2 = (AC)$ $\quad \vec{AC} = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$ $\quad g_2: \begin{cases} x \overset{(1)}{=} 2 - 7\lambda \\ y \overset{(2)}{=} 3 - \lambda \end{cases}$ $\lambda \in \mathbb{R}$

$\underset{(1)}{\lambda =} \frac{2-x}{7} \underset{\text{in }(2)}{\Rightarrow} \quad y = 3 - \left(\frac{2-x}{7}\right) \Rightarrow y = \frac{19}{7} + \frac{x}{7}$

$g_2: \quad y = \frac{x}{7} + \frac{19}{7}$

$g_2: \quad x - 7y + 19 = 0$ Koordinatengleichung

2 $\quad g_4 \perp g_1 \Rightarrow \vec{n}_4 = \lambda \begin{pmatrix} -3 \\ -9 \end{pmatrix} \underset{\lambda = -\frac{1}{3}}{=} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\quad \vec{u}_4 \cdot \vec{n}_4 = 0 \Rightarrow \vec{u}_4 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$\Rightarrow$ Parametergleichung von $g_4$ $\quad \begin{cases} x = -3\lambda \\ y = \lambda \end{cases}$ $\lambda \in \mathbb{R}$

$(0|0) \in g_4$

Koordinatengleichung $\Rightarrow$ $x + 3y = 0$ $\quad g_4$

3 $\quad g_5 \perp g_2 \Rightarrow \vec{n}_5 = \lambda \begin{pmatrix} -7 \\ -1 \end{pmatrix} \underset{\lambda = -1}{=} \begin{pmatrix} 7 \\ 1 \end{pmatrix} \Rightarrow 7x + y + c = 0$ ist Parametergleichung von $g_5$

$B = (-1|-6) \in g_5$ $\quad -7 - 6 + c = 0 \Rightarrow c = 13$

Koordinatengleichung $\quad 7x + y + 13 = 0$ $\quad g_5$

4 $\quad g_6 \perp g_3 \Rightarrow \vec{n}_6 = \lambda \begin{pmatrix} -4 \\ 8 \end{pmatrix} \underset{\lambda = -\frac{1}{4}}{=} \lambda \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow x - 2y + c = 0$ $(g_6)$ ist Parametergleichung

$A = (2|3) \in g_6 \Rightarrow 2 - 6 + c = 0 \Rightarrow c = 4$

$x - 2y + 4 = 0$ Koordinatengleichung

$\Rightarrow y = \frac{x+4}{2} = \frac{x}{2} + 2$ ist Funktionsgleichung von $g_6$

5 $\quad \delta(O, g_1) = \left| \frac{-3}{\sqrt{3^2+1^2}} \right| = \frac{3}{\sqrt{10}}$

$\delta(O, g_2) = \left| \frac{19}{\sqrt{1+7^2}} \right| = \frac{19}{\sqrt{50}} = \frac{19\sqrt{2}}{10}$

$\delta(O, g_3) = \left| \frac{8}{\sqrt{2^2+1}} \right| = \frac{8}{\sqrt{5}}$

$\delta(O, g_4) = \left| \frac{0}{\sqrt{1+3^2}} \right| = 0$

$\delta(O, g_5) = \left| \frac{13}{\sqrt{50}} \right| = \frac{13}{\sqrt{50}} = \frac{13\sqrt{2}}{10}$

$\delta(O, g_6) = \left| \frac{4}{\sqrt{5}} \right| = \frac{4}{\sqrt{5}}$

6 $\quad \delta(D, g_1) = \left| \frac{3 \cdot 2 - (-2) - 3}{\sqrt{3^2+1^2}} \right| = \frac{5}{\sqrt{10}} = \frac{\sqrt{10}}{2}$

$\delta(D, g_2) = \left| \frac{35}{\sqrt{50}} \right| = \frac{35}{\sqrt{50}} = 7\frac{\sqrt{2}}{2}$

$\delta(D, g_3) = \frac{10}{\sqrt{5}} = 2\sqrt{5}$

$\delta(D, g_4) = \left| \frac{-4}{\sqrt{10}} \right| = \frac{4}{\sqrt{10}}$

$\delta(D, g_5) = \left| \frac{25}{\sqrt{50}} \right| = \frac{5\sqrt{2}}{2}$

$\delta(D, g_6) = \left| \frac{10}{\sqrt{5}} \right| = 2\sqrt{5}$

7| $F_{\Delta ABC} = \frac{1}{2} \cdot$ Basis $\cdot$ Höhe , Basis $\overset{z.B.}{=} \underbrace{\|\overrightarrow{AB}\|}_{\sqrt{90}} \Rightarrow$ Höhe $= \underbrace{\delta(C; (AB))}_{\frac{|3\cdot(-5)-2-3|}{\sqrt{10}} = 2\sqrt{10}}$

$= \frac{1}{2} \cdot \sqrt{90} \cdot 2\sqrt{10} = \sqrt{900} = \underline{\underline{30\ FE}}$ (Flächeneinheiten)

8.| $E = g_5 \cap g_6$ $\begin{cases} 7x + y + 13 = 0 & g_5 \\ x - 2y + 4 = 0 & g_6 \end{cases} \overset{\text{lösen}}{\Longrightarrow} \begin{cases} x = -2 \\ y = +1 \end{cases}$ $\underline{E = (-2|1)}$

$F = g_3 \cap g_6$ $\begin{cases} 2x + y + 8 = 0 & g_3 \\ x - 2y + 4 = 0 & g_6 \end{cases} \overset{\text{lösen}}{\Longrightarrow} \begin{cases} x = -4 \\ y = 0 \end{cases}$ $F = (-4|0)$

$G = g_2 \cap g_5$ $\begin{cases} x - 7y + 19 = 0 & g_2 \\ 7x + y + 13 = 0 & g_5 \end{cases} \overset{\text{lösen}}{\Longrightarrow} \begin{cases} x = -\frac{11}{5} \\ y = \frac{12}{5} \end{cases}$ $G = \left(-\frac{11}{5}, \frac{12}{5}\right)$

$H = g_1 \cap g_4$ $\begin{cases} 3x - y - 3 = 0 & g_1 \\ x + 3y = 0 & g_4 \end{cases} \Rightarrow \begin{cases} x = \frac{9}{10} \\ y = -\frac{3}{10} \end{cases}$ $H = \left(\frac{9}{10} \middle| \frac{-3}{10}\right)$

9| $F_{\Delta FGO} = \frac{1}{2} \cdot$ Basis $\cdot$ Höhe mit Basis $= \delta(G; x\text{-Achse}) = \frac{12}{5}$

und Höhe $= \|\overrightarrow{OF}\| = 4$

$= \frac{1}{2} \cdot \frac{12}{5} \cdot 4$

$= \frac{24}{5} = \underline{4{,}8\ FE}$

10| $\overrightarrow{EK} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $g_7 = (EK)$ : $\begin{cases} x = 1 + 3\lambda \\ y = -\lambda \end{cases}$ Parametergleichung

$\Rightarrow$ $x + 3y - 1 = 0$ ist Koordinatengleichung von $g_7$

11| $I = g_4 \cap g_6$ $\begin{cases} x + 3y = 0 \\ x - 2y + 4 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{-12}{5} \\ y = \frac{4}{5} \end{cases}$ $I = \left(\frac{-12}{5} \middle| \frac{4}{5}\right)$

$J = g_4 \cap g_5$ $\begin{cases} x + 3y = 0 \\ 7x + y + 13 = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{39}{20} \\ y = \frac{13}{20} \end{cases}$ $J = \left(\frac{-39}{20} \middle| \frac{13}{20}\right)$

12| $g_4: \ x + 3y = 0$ $\vec{n}_4 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \vec{n}_7 \Rightarrow g_4 \parallel g_7$

$g_7: \ x + 3y - 1 = 0$ weil die Geraden denselben senkrechten Vektor aufweisen

13| $F_{\Delta EIJ} = \frac{1}{2} \cdot$ Basis $\cdot$ Höhe mit Basis $\overset{z.B.}{=} \|\overrightarrow{IJ}\| = \frac{3}{20}\sqrt{10}$

und Höhe $= \delta(E; g_4) = \frac{|-2+3|}{\sqrt{10}} = \frac{1}{\sqrt{10}}$

$= \frac{1}{2} \cdot \frac{3}{20}\sqrt{10} \cdot \frac{1}{\sqrt{10}} = \underline{\frac{3}{40}\ FE}$